THE PROVISION OF ACCURATE IMAGES WITH DYNAMIC GEOMETRY

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Results of a study involving pre-constructed, web-based, dynamic geometry sketches in activities at the secondary school level revealed that the provision of accurate images is an issue. Many students do not automatically understand that an onscreen image is accurate. Others inadvertently create a special case by dragging, then generalise from this static but unsuitable model. The research suggests that, for students to reap the benefits of working with an accurate image, they need to set aside, during the exploration phase, the traditional attitude of suspicion towards diagrams, and to recognize that dynamic geometry diagrams offer valuable and accurate visual evidence.

INTRODUCTION

The study that informs this paper was undertaken to evaluate the benefits and limitations of the use of pre-constructed, web-based, dynamic geometry sketches in activities related to proof at the secondary school level. From the analysis of the data two main themes were identified: 1) the relationship between the activities and the development of geometric thinking skills, and 2) the relationship between the design of the materials and the exploration process. Underlying these was an important sub-theme--how students responded to a visually accurate image.

Many researchers have recommended an increased emphasis on the use of visual reasoning in mathematics (cf., Duval, 1998; Goldenberg et al, 1998; Presmeg, 1999; Dreyfus, 1991). With respect to computer images, Sutherland and Balacheff (1999) reported that visual images displayed on computer screens allow students to gain access to mathematical knowledge by “rendering more visible the nature of the objects with which a student is engaging” (p. 2), and in a 1986 study of visualisation in high school students, Presmeg found that dynamic imagery--although used by only a few "visualisers"-- was effective in helping students generalise. The implication is that visually accurate images are beneficial in helping students understand geometric ideas.

Visual geometric images whether printed or onscreen, fall into three categories: 1) special case, 2) general case, and 3) inaccurate. One might argue that the last category is unnecessary, however, an examination of most secondary texts will reveal many diagrams that include measurements but are not drawn to scale. In keeping with tradition, teachers warn their students that the diagrams are not necessarily accurate and that they are to focus only on making logical deductions. The results of this study suggest that the ‘diagram bias’ thus created acts as a roadblock when students examine a pre-constructed, and ‘accurate’ (i.e., to within a measurable error), dynamic sketch.

Pre-constructed sketches

Pre-constructed sketches created with *Cabri Géomètre* (Baulac, Bellemain, and Laborde, 1992), or *The Geometer's Sketchpad* (Jackiw, 1991) as well as pre-constructed, web-based sketches created with *JavaSketchpad* (Jackiw, 1998), or Cinderella (1999, Richter-Gebert and Kortenkamp) can be used as an alternative to having students construct their

own dynamic diagrams. In all pre-constructed dynamic sketches, points can be dragged; pre-set relationships, such as measurements and ratios, update as a consequence of dragging; and action buttons to hide or show details, to move and to animate objects can be included. Web-based sketches created with *JavaSketchpad* do not permit the user to construct or delete objects; however, those designed with Cinderella can provide options for constructing a limited number of objects. Angles and lengths in these sketches are represented accurately to within a small error, unlike those in textbook diagrams.

Interpretation and theoretical framework

Whether a pre-constructed sketch is web-based or not, it presents a geometric situation to the student in visual format. Since the creator of an image knows details that are hidden from an ordinary viewer, interpreting a pre-constructed sketch is similar to interpreting a picture that someone else has drawn. Measurements or measurement tools are provided, but students must apply their own organisation to the information, and draw conclusions about how items are connected—a difficult task because mathematical pictures and diagrams contain a great deal of information represented in a concise but "nonsequential" format (Goldenberg, Cuoco and Mark, 1998).

Dynamic sketches include several options for motion including animation capabilities and the dragging provision, which allows the student to explore an object in motion, at a controlled speed. In 1998, Arzarello, Micheletti, Olivero, Robutti, Paola, and Gallino classified modalities of dragging as: “Dragging test”, “wandering dragging”, and “lieu muet” (dummy locus). They found that students who produced good conjectures made use of "lieu muet" dragging, a purposeful mode which "can be seen as a wandering dragging which has found its path" (p. 37).

Extensive studies of Cabri have shown that a geometry problem cannot be solved simply by perceiving the onscreen images, even if these are animated. The student must bring some explicit mathematical knowledge to the process (p.32). That is, an intuition about a generalization involves more than observed evidence (Fischbein, 1987). This study, on the use of pre-constructed, dynamic geometry sketches, found that the basic task of perceiving detail, which involves noticing lengths, angles, measurements, labels, markings, then noticing the change in these as a consequence of dragging, is difficult for many students. I contend that this is in part due to traditional but limiting attitudes towards diagrams that students bring to the dynamic environment.

DESCRIPTION OF THE STUDY

The research used a case study approach and multiple sources of information -- observation field notes, videotape, audiotape, a student questionnaire, and interviews with teachers. Collected data was transcribed, then analysed by coding, developing categories, describing relationships, and applying simple statistical tests where appropriate.

The transcripts were coded in several ways to allow analysis of student actions and thinking, and to link these to particular labsheet questions or sketch features. During this process, students’ uses of and responses to the visual images were examined (see Sinclair, 2001 for further detail).

Three mathematics classes from two different secondary schools participated in this study. The 69 students were enrolled in the Ontario grade twelve advanced mathematics

program (replaced in 2002), which covered topics in algebra, geometry, analytic geometry and trigonometry (Curriculum guideline, 1985). The study focused on congruence and parallelism, the first section in the geometry unit. Although the students had done introductory work on deductive geometry related to congruence and parallelism in grade 10 and on similarity in grade 11, none had worked with dynamic geometry software.

Three 75 minutes sessions or four 45 minutes sessions were held with each class. During this time, students worked in pairs on four tasks. An additional task was done as a whole class activity. In each class, several pairs were studied in more depth by audiotaping or videotaping their activities.

JavaSketchpad, was used to prepare four web-based, dynamic geometry sketches for student pairs to explore during the sessions, two extra sketches for those who finished early, and one sketch for a group discussion. The labsheet that accompanied each sketch provided directions for opening and manipulating the sketch, a statement of the problem, and questions related to the task.

Problems chosen as the basis for the web-based sketches were similar in difficulty to those in the student text, *Mathematics: Principles and Process, Book 2* (Ebos, Tuck, and Schofield, 1986) and related to triangles and quadrilaterals.

Each of the sketches supported the possibility of arriving at a solution from a transformation perspective as well as from a straightforward application of congruency theorems. The intention was to allow students to use symmetry considerations, a) to visually confirm or negate conjectures, and b) to develop a new perspective on geometric relationships.

In the pre-study interview the three study teachers identified difficulties that their students experience in the geometry strand. For example, teachers mentioned that students constructing congruency proofs frequently select sides or angles that do not correspond to one another, or, in fact, do not even belong to the subject triangles. They noted that this problem usually occurs when figures overlap or are presented in rotated, reflected, or translated form. These student difficulties reveal an inability to “see” each overlapping figure separately or to mentally transform a figure to a new orientation to compare it with another. To address these difficulties, sketches included action buttons or provisions to highlight particular figures, to toggle details on and off, and to rotate or reflect shapes so that they could be superimposed, or viewed from the same orientation.

Overview of a session task

A very brief overview of one task is included here to help the reader understand the context for the discussion.

Day 2, task 1.

This task gave students the opportunity to apply properties of parallel lines and to investigate a problem using a rotation.

The triangles to be proven congruent were coloured to attract student attention. When a vertex of quadrilateral ABCD was dragged, AD and BC appeared to remain equal and parallel, as did AB and DC. When the "Show Given Information" button was used,

students could deduce that ABCD was indeed a parallelogram since opposite sides were marked equal and measurements were given.

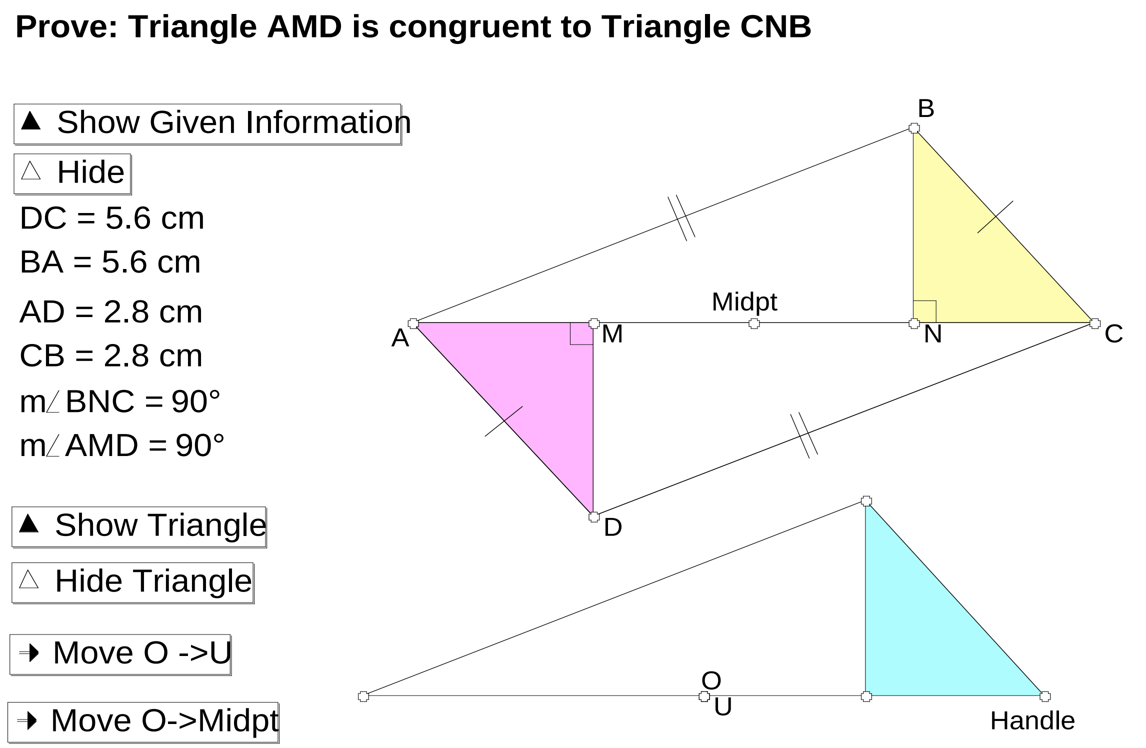

Figure 1: Day 2, task 1—After selecting: "Show Given Information," "Show Triangle" and “Move O→ U.”

It was expected that students would use ASA (angle, side, angle congruency theorem) to prove that △AMD and △BNC were congruent: AD = BC (given), ∠DAM = ∠BCN (parallel line law), and ∠MDA = ∠NBC; however, students could also investigate the relationship between the two by superimposing an additional given triangle over △ABC and then rotating it to fit over △CDA. This movable triangle was a tool for testing whether △AMD and △BNC were congruent; however, it could also be used to demonstrate the fact that congruent triangles have congruent altitudes (i.e., △ABC and △ADC are congruent, which implies that AM must equal BN). Questions on the labsheet such as: “What do you notice about the new triangle?” and “How can the information provided by these images be used to explain why DM = BN?” were aimed at helping students notice and address the information provided in the sketch.

DISCUSSION

The particular responses (or non-responses) to the provision of an accurate visual image were related to three broad categories: noticing details, use of dragging, and entertaining alternative methods.

Noticing details

The following (unconnected) comments show that students noticed details in the sketches. (Note: all students are identified by pseudonyms.)

Doug: Angle BED--hey!…Angle BED is 72.455.

Katy: Um, uh the angle shadings They're the same angles. Yeah, I would say that. The angle shadings mean that they're congruent angles. So, congruent sides and congruent angles.

Dave: They match. It matched it with the other one. It shows us that they're congruent.

Bea: Oh, so the yellow and the purple

Familiar markings and colour drew attention--students noticed items that were coloured or marked and sometimes missed those that weren't. Colour was also used as a simple and effective means of referencing objects in discussion as shown by Bea's comment. On the other hand, despite Doug's comment, the transcripts show that measurements were often ignored.

It is not clear whether some students did not notice the measurements (in *JavaSketchpad* lengths and angles are in a list and not attached to the object), or whether they were so attuned to the "rules" of deductive geometry that they did not expect to use measurement data. The ability to display an accurate image is commonly assumed to be a benefit of dynamic geometry software--it seems reasonable to conclude that the task of noticing and interpreting relationships between objects is easier if figures are drawn to scale. However, the study results showed that many students either do not realize or ignore the fact that the onscreen image is accurate.

The tendency of study students to gloss over measurements is in stark contrast to their awareness of colour and markings. It is of concern because the ability to explore how a figure has changed requires focused attention to details that update under the operation of dragging.

Use of dragging

Although initially intrigued by the ability to drag points, study students usually stopped dragging after a short time and concentrated on interpreting a static figure. This led to mixed results.

Some students treated the onscreen image as if it were a pencil sketch--as if the diagram represented objects and their relationships, but was not drawn to scale. For example, two above average students made the following (unconnected) comments:

Barb: Maybe cause it's slanted you can't tell it's a square.

Sue: If this is equilateral these sides would have to be equal. [In this instance, the triangle was clearly not equilateral].

I hypothesised that such responses might stem from prior use of textbook diagrams. Geometry teachers frequently warn their students not to make conclusions based on the appearance of diagrams that are not necessarily accurate. These students used their knowledge of deductive theorems to correctly solve the problems, but gained nothing from being able to use an accurate model.

In the following example, abandoning dragging led to an erroneous conclusion. Doug and Sal were looking at a triangle in Day 1, task 2 (not shown). Angle BEA may have been very close to a right angle on their diagram, but if they had dragged the sketch they would have seen it change.

Doug: .. BEA--angle E is 90 degrees..

Sal: There's no thing [referring to the symbol for a 90 degree angle]

Doug: Well, you can't see it.

Sal: That's right

Doug: Well, I'm thinking this is an[sic]--cause it looks like it, right?

In this case, Doug and Sal, two average students, actually did treat the sketch as accurate! Along with some other students they persisted in drawing conclusions based on what the angles or sides looked like. Certainly many informal conjectures suggest themselves to mathematicians because they "look like" they are true. Some withstand further investigation; others prove false. The students' problem was not in basing a guess on the visual evidence before them but in failing to realise that they were observing a specific case.

Entertaining alternate methods

Some student responses draw attention to the fact that we often focus on methods that are more suited to symbolic rather than visual analysis. This tendency forces the student to turn away from the image medium to compose a proof. For example, in Day2, task 1 (see Fig 1), the sketch allowed students to use rotation to explain why two segments were equal, instead of deducing the result via a triangle congruency proof. The question was: "How can the information provided by these images be used to explain why DM equals BN?" Students intuitively understood that when the triangle was rotated, DM would fall on BN.

Clara: Because the triangle fits--the triangle fits both.

Despite this comment Clara did not follow up with a step-by-step analysis. She felt that the result needed no further explanation, but she was unable to compose a written transformation-based proof. Instead she used a traditional congruency proof.

If Clara wanted to justify her conclusion without abandoning the image that made the congruency clear, what language would she use? I do not think most teachers are able to step back from their deductive geometry experiences to provide a simple, clear proof that uses transformation concepts and takes advantage of the visual reality offered by an onscreen image.

Examples such as this also highlight students' unfamiliarity with describing visual information in precise terms. We cannot intuit if we do not perceive and I contend that students are not taught to perceive visual details—they are taught to select information from diagrams—even if this information is false to the eye.

CONCLUSION

In their 1996 summary analysis of research on computer-based learning environments in mathematics, Balacheff and Kaput note that one of the ways in which the computer makes its primary impact is by "changing the relationships between learners and the subject matter and between learners and teachers—by introducing a new partner" (p. 495). The resulting environment is didactically complex. In addition to the usual teacher-student interactions, there are interrelationships among the student, the computer and the task (Sutherland & Balacheff, 1999). The nature of this student-computer interchange is shaped by the unique characteristics of the software and its objects. This paper has briefly presented how the accuracy of pre-constructed, dynamic geometry sketches affects and is affected by the experience of the secondary school geometry student.

The study students who treated dynamic sketches like textbook models missed visual evidence that might have provided support for a deeper understanding of geometric relationships. Having always been provided with diagrams that carefully displayed a general case, some students did not recognize the pitfalls of creating and analyzing a special case. And when students were able to bring only traditional proof techniques to bear on dynamic problems they failed to experience the true power of dynamic software.

As educators we need to be aware of the biases that our students bring to their work. In the case of accurate and interactive images we need to explicitly focus students' attention on the differences between textbook diagrams and dynamic geometry sketches, and help them find ways to mine the benefits of visual reality.

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